The real-time dynamics of lattice field theories via machine learning

Yukari Yamauchi

arXiv:2101.05755 with Scott Lawrence

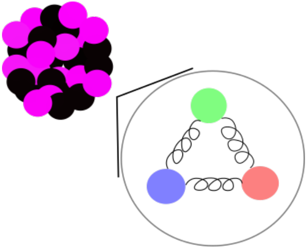
10/28/2021, Deep Learning and Physics Online, Japan

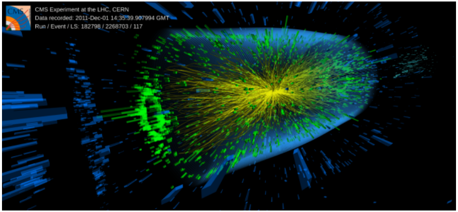
University of Maryland
University of Colorado Boulder

Contents

Quantum Chromodynamics

The theory of quarks and gluons:





Tom McCauley/CMS/CERN

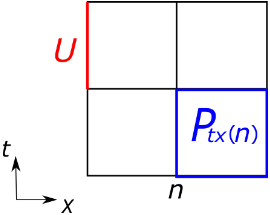
$$S_{QCD} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Lattice QCD

Non-perturbative method to compute Feynman path integral

Discretize spacetime

cutoff $p < \frac{1}{a}$



Links U take values $g \in SU(3)$

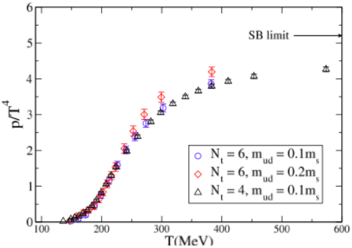
Lattice action (Euclidean, gauge part)

$$S_{\mathrm{W}} = \frac{2}{g_s^2} \sum_{n\in\Lambda} \sum_{\mu<\nu} \mathrm{Re}\,\mathrm{Tr}[1 - P_{\mu\nu}(n)] + O(a^2)$$

Path intergal for expectation values

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}[U] e^{-S(U)} \mathcal{O}(U)}{\int \mathcal{D}[U] e^{-S(U)}}$$

via Markov chain Monte Carlo sampling

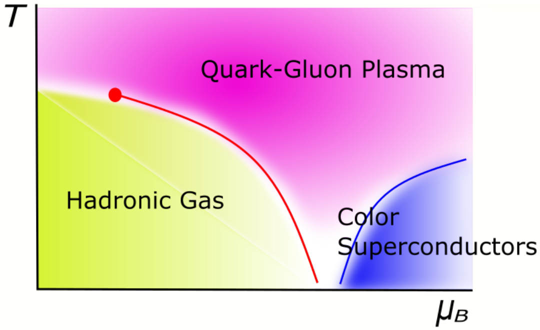


S. Basak, et al. arXiv:0910.0276

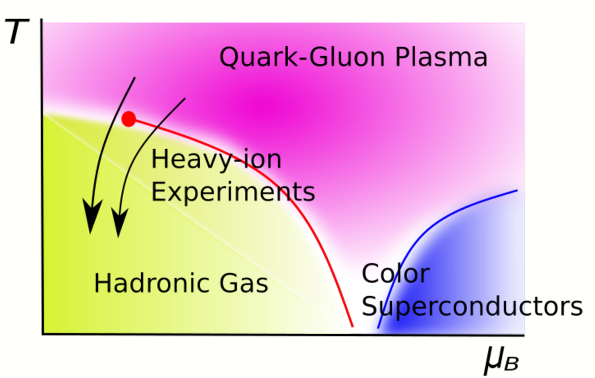
Sign problems

With naive lattice QCD, exponential (in volume) costs to compute

Finite μ_B and **real-time** observables



Shear viscosity of QCD



The shear viscosity η near T_c[1]

$$\frac{1}{4\pi} < \frac{\eta}{s} < \frac{2.5}{4\pi} \text{ for } T_c < T \leq 2T_c$$

[1] H. Song, S. A. Bass, U. Heinz, T. Hirano, and C. Shen, Phys. Rev. Lett. 106, 192301 (2011)

Inputs to hydrodynamics

Hydrodynamic fields $\epsilon(\vec{x}), u_\mu(\vec{x})$:

$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + T^{(1)}_{\mu\nu} + \cdots$$

$$T^{(0)}_{\mu\nu} = (\epsilon + P)u_\mu u_\nu + P\, g_{\mu\nu}, \quad T^{(1)}_{\mu\nu} = F(\partial_\mu \epsilon,\ \partial_\mu u_\nu;\ \eta,\ \zeta)$$

(P: pressure, η: shear viscosity, ζ: bulk viscosity)

Shear viscosity η:

- From T_{01} correlator:

$$\int_V d\vec{x}\ e^{i\vec{k}\cdot\vec{x}} \langle\phi(\beta)|\,[T_{01}(t,\vec{x}), T_{01}(0,0)]\,|\phi(\beta)\rangle \sim e^{-\frac{\eta(\beta)k^2}{\epsilon}t}$$

- From T_{12} correlator:

$$\eta(\beta) = \frac{1}{T}\int_V dx\ \int_0^\infty dt\ \langle\phi(\beta)|\,[T_{12}(x,t), T_{12}(0,0)]\,|\phi(\beta)\rangle$$

Classical computers → **Real-time sign problems**

Quantum computers → **No** sign problems, but only small computers exist

What can classical computers do?

Euclidean correlators[2] $\rightarrow$ Error bar on η

Minkowski correlators $\rightarrow$ Real-time sign problem

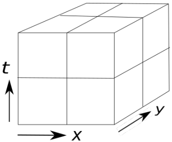
[2] H. Meyer, Phys.RevD 76 (2007) 101701

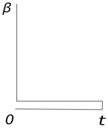
Real-time sign problem

Goal: compute $\langle \phi(\beta)| \, [\mathcal{O}(t,\vec{x}), \mathcal{O}(0,0)] \, |\phi(\beta)\rangle$

Method: Lattice QCD via Markov chain Monte Carlo sampling

$$\langle \mathcal{O}(t,\vec{x}) \rangle = \frac{1}{Z} \int \mathcal{D}[\psi, U] e^{-S} \mathcal{O}(t,\vec{x})$$



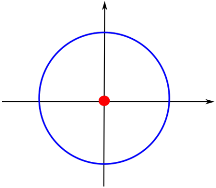


The action S is complex $\rightarrow$ define "quenched distribution" $e^{-\operatorname{Re} S}$

$$\langle \mathcal{O} \rangle = \frac{\left\langle e^{-i \operatorname{Im} S} \mathcal{O} \right\rangle_{e^{-\operatorname{Re} S}}}{\left\langle e^{-i \operatorname{Im} S} \right\rangle_{e^{-\operatorname{Re} S}}}$$

Especially the **"average sign"** is challenging:

$$\langle \sigma \rangle = \left\langle e^{-i \operatorname{Im} S} \right\rangle_{e^{-\operatorname{Re} S}} \propto a^V, \quad |a| \leq 1$$



Toward solving some sign problems[4]

- Manifold deformation method[3]
- Perfect manifolds ($\langle \sigma \rangle = 1$) in terms of "complex normalizing flows"
- Perfect manifolds (and thus complex normalizing flows) exist!
- How do we find perfect manifolds or complex normalizing flows? Machine learning?

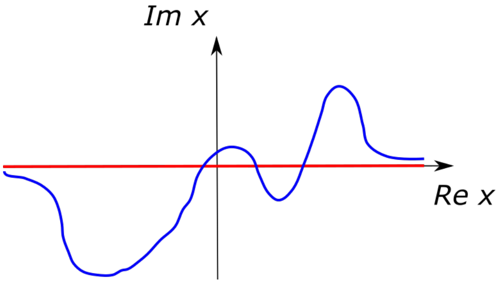
[3] For review, see A. Alexandru et al., arXiv:2007.05436

[4] S. Lawrence and YY, arXiv:2101.05755

Manifold deformation

One candidate for solving the sign problem is the **Manifold Deformation**

Example: Action $S(x) \rightarrow$ Path integral $\int dx\ e^{-S(x)}$



$$\langle\sigma\rangle_{\mathcal{M}} = \frac{\int_{\mathcal{M}} e^{-S}}{\int_{\mathcal{M}} e^{-\operatorname{Re} S}} \overset{?}{>} \frac{\int_{\mathbb{R}} e^{-S}}{\int_{\mathbb{R}} e^{-\operatorname{Re} S}} = \langle\sigma\rangle_{\mathbb{R}}$$

1D example[5]

The action: $S = x^2 + 2i\alpha x$

$$\langle \sigma \rangle = e^{-\alpha^2}$$

Change the integration contour to

$$z = t - i\alpha, \quad -\infty < t < \infty$$

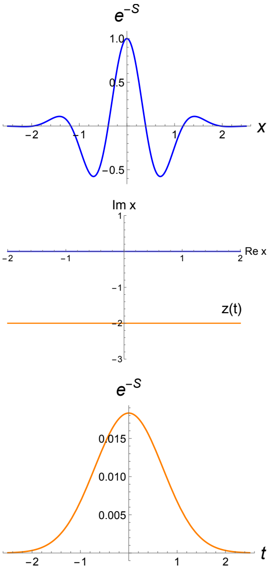
The action on —

$$S(z(t)) = -t^2 - \alpha^2$$

The average sign:

$$\langle \sigma \rangle = \frac{\int dt\ e^{-\operatorname{Re} S - i \operatorname{Im} S}}{\int dt\ e^{-\operatorname{Re} S}} = 1$$

No sign problem!!



[5] S. Lawrence, Thesis, arXiv:2006.03683

Perfect manifold and normalzing flow

On a **perfect manifold** $z(t) \subset \mathbb{C}$,

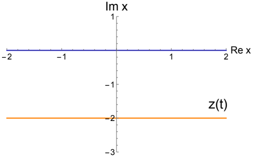
The Boltzmann factor in t:

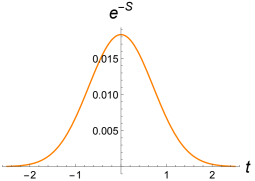
$$f(t) = \frac{dz}{dt} e^{-S(z(t))}$$

is **real and positive**

$$\langle \sigma \rangle = \frac{\int dt\ e^{-S}}{\int dt\ e^{-\mathrm{Re}\, S}} = 1$$

$z(t)$ as a map from $\mathbb{R}$ to $\mathbb{C}$:





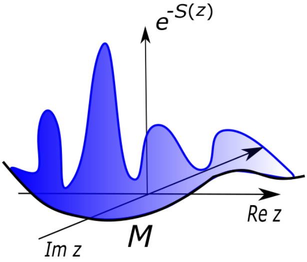
$$dz\ e^{-S(z)} = dt\ \frac{dz(t)}{dt}\ e^{-S(z(t))} = dt\ \mathcal{N} e^{-t^2/2}$$

(When S is real, called normalizing flow[6] $\mathbb{R}$ to $\mathbb{R}$)

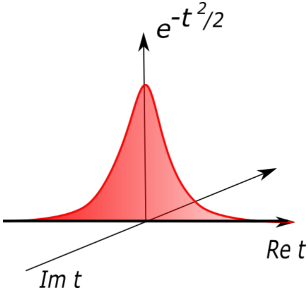
Perfect Manifold exists ↔ Complex normalizing flow $z(t)$ exists

[6]M. Albergo et al. Phys. Rev. D 100, 034515(2019)
K. A. Nicoli, et al. Phys. Rev. E 101, 023304(2020)

Complex normalizing flow



Map
$\leftrightarrow$
$z(t)$



Expectation values:

$$\langle \mathcal{O} \rangle_{\mathcal{M}} = \frac{\int_{\mathcal{M}} dz\ e^{-S(z)} \mathcal{O}(z)}{\int_{\mathcal{M}} dz\ e^{-S(z)}} = \frac{\int_{\mathbb{R}} dt\ e^{-t^2/2}\ \mathcal{O}(z(t))}{\int_{\mathbb{R}} dt\ e^{-t^2/2}}$$

$$\stackrel{?}{=} \frac{\int_{\mathbb{R}} dz\ e^{-S(z)} \mathcal{O}(z)}{\int_{\mathbb{R}} dz\ e^{-S(z)}} = \langle \mathcal{O} \rangle$$

when 3 conditions on $\mathcal{M}$ are met.

Constraints on manifolds[7]

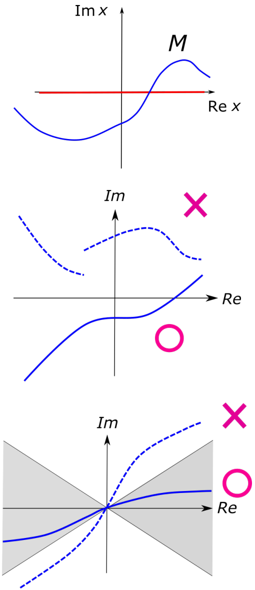
Manifolds give the correct $\langle \mathcal{O} \rangle$

$$\langle \mathcal{O} \rangle = \frac{\int_{\mathbb{R}} dx\ e^{-S(x)} \mathcal{O}(x)}{\int_{\mathbb{R}} dx\ e^{-S(x)}} = \frac{\int_{\mathcal{M}} dz\ e^{-S(z)} \mathcal{O}(z)}{\int_{\mathcal{M}} dz\ e^{-S(z)}}$$

when:

- The manifold (—) is a continuous manifold
- The manifold (—) is in "asymptotically safe" region
- Both e^{-S} and $e^{-S}\mathcal{O}$ are holomorphic functions in the region between (—) and (—)

$\rightarrow$ **Cauchy's integral theorem!**



[7] A. Alexandru et al., Phys. Rev. D. 98, 034506(2018)

Do perfect manifolds exist?

Perfect manifolds exist $\leftrightarrow$ Normalizing flows exist

Do perfect manifolds exist?

If so, can we find them?

If so can use them?

There exist perfect manifolds at least for bosonic theories!

A conjecture on normalizing flows

Type of action: action $S(\vec{z})$ which is finite except at $|z| = \infty$

When with **NO** sign problems (S is real)

Fact: Normalizing flows exist. ($\mathbb{R}^N \to \mathbb{R}^N$)

Conjecture:
Normalizin flows are analytic functions of the parameters of the action.

Example: Scalar field theory $S(z; M, \Lambda) = z_i\, M_{ij}\, z_j + g\, \Lambda_i\, z_i^4$ on N sites

The map: $\left(\det \frac{d\vec{z}(\vec{t}; M, \Lambda)}{d\vec{t}}\right)\; e^{-S(z(\vec{t}; M, \Lambda))} = \mathcal{N}\; \left(e^{-t^2/2}\right)^N$

Perturbative map in weak g:

$$z_i(\vec{t}; M, \Lambda) = z_i - g\left(\sum_j \tfrac{1}{2}\, M^{-1}_{ij}\, \Lambda_j\, t_j^3 + \tfrac{3}{4}\, M^{-1}_{ij}\, M^{-1}_{jj}\, \Lambda_j\, t_j\right)$$

(analytic in M, Λ except at $\det M = 0$)

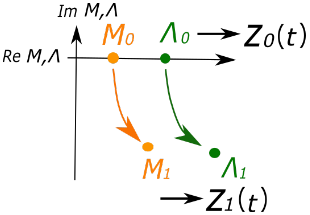
Perturbative map in strong g is analytic in M, Λ except at $\Lambda = 0$

Existence of perfect manifolds[8]

Conjecture:
Normalizing flows are analytic functions of the parameters of the action, when $M, \Lambda \in \mathbb{R}$.

Conjecture implies:

Perfect manifolds exist for $M, \Lambda \in \mathbb{C}$



Caveat:
When manifolds intersect with singularity of S, perfect manifolds are not guaranteed to exist

$SU(3)$ **Yang-Mills in Minkowski** – O

Fermionic theories – ?

Can we find them? If so can use them?

[8]S. Lawrence and YY, arXiv:2101.05755

Example with scalar field theory[9]

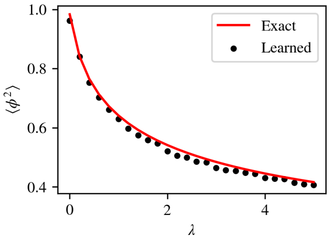
To estimate $\langle \mathcal{O} \rangle_S$ for the action S, let us define $S' = S + \alpha\, \mathcal{O}$.

A perturbing map $\vec{y}(x)$ from $S'(x + \alpha\, \vec{y}(x))$ to $S(x)$ satisfies

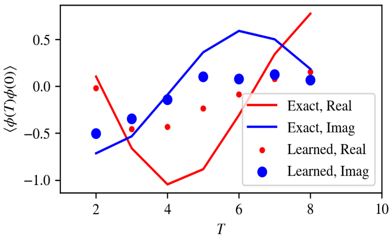
$$\nabla \cdot \vec{y}(x) - \vec{y}(x) \cdot \nabla S(x) - \mathcal{O}(x) + \langle \mathcal{O} \rangle_S = 0$$

Solve the ODE for $\vec{y}(x)$ and $\langle \mathcal{O} \rangle_S$ via machine learning:

$$C(w, \langle \mathcal{O} \rangle_S) = \sum_x |\nabla \cdot \vec{y}_w(x) - \vec{y}_w(x) \cdot \nabla S(x) - \mathcal{O}(x) + \langle \mathcal{O} \rangle_S|^2$$



$0 + 1\text{d},\ m = 0.5,\ N_\beta = 10,\ N_T = 0$



$0 + 1\text{d},\ m = 0.5, \lambda = 0.5, N_\beta = 2$

[9] S. Lawrence and YY, arXiv:2101.05755

Ongoing work

Establish numerical methods to find flows/manifolds

Normalizing flows:

$$\left(\det \frac{d\vec{z}(\vec{t};M,\Lambda)}{d\vec{t}}\right)\; e^{-S(z(\vec{t};M,\Lambda))} = \mathcal{N}\; e^{-\vec{t}\cdot\vec{t}/2}$$

1. Represent a normalizing flow with a neural network (parameters $\vec{w}$.
2. Sample $\vec{t}$ from Gaussian distribution.
3. The loss function is:

$$L(\vec{w}) = \sum_{\vec{t}} \left| \left(\det \frac{d\vec{z}(\vec{t};M,\Lambda)}{d\vec{t}}\right)\; e^{-S(z(\vec{t};M,\Lambda))} - \mathcal{N}\; e^{-\vec{t}\cdot\vec{t}/2}\right|^2$$

4. Train the neural network with the loss function.

Thank you!